Trigonometric Identity IV

$$\tan^2\theta \equiv \sec^2\theta - 1$$

This is a helpful equation used to relate the functions tangent (otherwise known as tan) and secant (otherwise known as sec). $\tan^2\theta$ is the same thing as $(\tan\theta)^2$, it is merely an easier way of writing it, the same is true for $\sec^2\theta$. The $\equiv$ symbol means "identical to" (i.e. tan squared theta is identical to sec squared theta minus one). This symbol means the relationship is always true, regardless of the value of θ. θ is a placeholder for an angle, and for this identity to work the angle must be the same for both tan and sec.

Proof

Starting with Trigonometric Identity I,

$$\sin^2\theta + \cos^2\theta \equiv 1$$

Dividing both sides by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} \equiv \frac{1}{\cos^2\theta}$$

Using Trigonometric Identity II, $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$

$$\tan^2\theta + \frac{\cos^2\theta}{\cos^2\theta} \equiv \frac{1}{\cos^2\theta}$$

Simplifying $\frac{\cos^2\theta}{\cos^2\theta}$

$$\tan^2\theta + 1 \equiv \frac{1}{\cos^2\theta}$$

Using our knowledge that $\sec\theta = \frac{1}{\cos\theta}$

$$\tan^2\theta + 1 \equiv \sec^2\theta$$

By re-arranging we find that

$$\tan^2\theta \equiv \sec^2\theta - 1$$

See also

- Cosecant, Secant and Cotangent
- Trigonometric Identity I
- Trigonometric Identity II
- Trigonometric Identity III

References

Attwood, G. et al. (2017). *Edexcel A level Mathematics - Pure - Year 2*. London: Pearson Education. pp.153-154.